On Weak McCoy Rings

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Abstract

In this note we introduce the notion of weak McCoy rings as a generalization of McCoy rings, and investigate their properties. Also we show that, if R is a semi-commutative ring, then R is weak McCoy if and only if $R[x]$ is weak McCoy.

1. Introduction

Throughout this paper, all rings are associative with identity. For a commutative ring R, McCoy [10] obtained the following result: If $f(x)g(x)=0$ for some non-zero polynomials $f(x), g(x)\in R[x]$, then $f(x)c=0$ for some non-zero $c\in R$. According to Nielsen [12], a ring R is called *right McCoy* whenever polynomials $f(x)$, $g(x)\in R[x]-\{0\}$ satisfy $f(x)g(x)=0$, there exists a non-zero $r\in R$ such that $f(x)r=0$. Left McCoy rings are defined similarly. If a ring is both left and right McCoy, we say that the ring is a *McCoy ring*. It is well known that commutative rings are always McCoy rings [10], but it is not true for non-commutative rings (see [12]).

Recall that a ring R is called:

reduced if $a^2=0 \Rightarrow a=0$, for all $a\in R$,

reversible if $ab=0 \Rightarrow ba=0$, for all $a,b\in R$,

symmetric if $abc=0 \Rightarrow acb=0$, for all $a,b,c\in R$,

semi-commutative if $ab=0 \Rightarrow aRb=0$, for all $a,b\in R$.

The following implications hold:

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reduced $\Rightarrow$ symmetric $\Rightarrow$ reversible $\Rightarrow$ semi-commutative.

Reversible rings are McCoy rings by [12]. But the converse is not true; there exists a non-reversible McCoy ring (see [12]).

Motivated by the above, as a generalization of McCoy rings, in this paper we introduce the notion of weak McCoy rings and investigate their properties and extend several known results relating to McCoy rings to a general setting.

For a ring R, we denote by $nil(R)$ the set of all nilpotent elements of R, by $N_*(R)$ the prime radical of R and by $M_n(R)$, $U_n(R)$ and $L_n(R)$ the $n\times n$ matrix ring over R, the $n\times n$ upper and lower triangular matrix rings over R respectively.

2. On Weak McCoy rings

Definition2.1. We say R is a *weak McCoy* ring if $f(x)g(x)\in nil(R[x])$ implies $f(x)c\in nil(R[x])$, for some non-zero $c\in R$, where $f(x)$ and $g(x)$ are non-zero polynomials in $R[x]$.

Remark 2.2. Since ab is nilpotent if and only if ba is nilpotent in a ring, hence the definition of weak McCoy rings is left-right symmetric.

Proposition 2.3. McCoy rings are weak McCoy.

Proof. Let R be a McCoy ring and $f(x)g(x)\in nil(R[x])$ for non-zero polynomials $f(x)$, $g(x)\in R[x]$. Then there exists $m,n\geq 1$, such that $(f(x)g(x))^n=(g(x)f(x))^m=0$, and $(f(x)g(x))^{n-1},(g(x)f(x))^{m-1}\neq 0$. If $f(x)g(x)=0$ or $g(x)f(x)=0$, then the result follows from the definition of McCoy rings. Assume $f(x)g(x)\neq 0\neq g(x)f(x)$ and $0=(f(x)g(x))^n=f(x)(g(x)f(x)\ldots f(x)g(x))=f(x)h(x)$.

If $h(x)=g(x)f(x)\ldots f(x)g(x)\neq 0$, then $f(x)c=0$ for some non-zero $c\in R$, since R is McCoy.

Let $h(x)=g(x)(f(x)g(x)\ldots f(x)g(x))=g(x)(f(x)g(x))^{n-1}=0$. Since $(f(x)g(x))^{n-1}\neq 0$ and R is McCoy, there exists $0\neq d\in R$ such that $g(x)d=0$. Therefore $f(x)c=0$ or

$g(x)d=0$ for some non-zero $c,d\in R$. Hence $f(x)c\in nil(R[x])$ or $dg(x)\in nil(R[x])$ for some non-zero $c,d\in R$. Therefore R is weak McCoy.

Proposition 2.4. Let R be a ring. Then $U_n(R)$ and $L_n(R)$ are weak McCoy for each $n\geq 2$.

Proof. Clearly $U_n(R)[x]\cong U_n(R[x])$ and for each $A=\begin{bmatrix} 0 & f_{12} & \cdots & f_{1n} \\ 0 & 0 & \cdots & f_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & 0 \end{bmatrix}\in U_n(R[x])$, $A^n=0$. Let $0\neq A=\begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ 0 & f_{22} & \cdots & f_{2n} \\ 0 & 0 & \ddots & \cdots \\ 0 & 0 & \cdots & f_{nn} \end{bmatrix}\in U_n(R[x])$. Then

$$A\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}=\begin{bmatrix} 0 & g_{12} & \cdots & g_{1n} \\ 0 & 0 & \cdots & g_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \text{ and } \left(A\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 1 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)^n=0.$$

Hence $U_n(R)$ is weak McCoy. By a similar argument one can show that $L_n(R)$ is weak McCoy.

Proposition 2.5. Let R and S be rings and ${}_RM_S$ a bimodule. Then $\begin{bmatrix} R & M \\ 0 & S \end{bmatrix}$ is a weak McCoy ring.

Proof. Similarly, as used in Proposition 2.4 one can prove it.

The following example shows that $U_n(R)$ and $M_n(R)$ are neither left nor right McCoy for some $n\geq 2$.

Example 2.6. Let R be a ring. We show that $U_4(R)$ and $M_4(R)$ are neither right nor left McCoy. Let $f(x)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}+\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}x$ and

$$g(x)=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}+\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}x\in U_4(R)[x]\subseteq M_4(R)[x]$$. Then $f(x)g(x)=0$.

If $f(x)A=0$, for some $A=\lfloor a_{ij}\rfloor\in M_4(R)$, then $0=\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}A=\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\0&0&0&0\\a_{31}&a_{32}&a_{33}&a_{34}\\0&0&0&0\end{bmatrix}$

and $0=\begin{bmatrix}0&-1&0&0\\0&0&0&0\\0&0&0&-1\\0&0&0&0\end{bmatrix}A=\begin{bmatrix}-a_{21}&-a_{22}&-a_{23}&-a_{24}\\0&0&0&0\\-a_{41}&-a_{42}&-a_{43}&-a_{44}\\0&0&0&0\end{bmatrix}$. Hence $A=0$ and $U_4(R)$ and $M_4(R)$ are not right McCoy. If $Bg(x)=0$ for some $B\in M_4(R)$, then by a similar way as above, we can show $B=0$. Therefore $U_4(R)$ and $M_4(R)$ are not left McCoy.

Definition 2.7. A ring R is called *right Ore* if given $a,b\in R$ with b regular there exist $a_1,b_1\in R$ with b_1 regular such that $ab_1=ba_1$. It is well-known that R is a right Ore ring if and only if the classical right quotient ring of R exists. We use $C(R)$ to denote the set of all regular elements in R.

Theorem 2.8. Let R be a right Ore ring with its classical right quotient ring Q. If R is weak McCoy then Q is weak McCoy.

Proof. Let $0\neq F(x)=\sum_{i=0}^{m}a_iu^{-1}x^i$ and $0\neq G(x)=\sum_{j=0}^{n}b_jv^{-1}x^j$ with $a_i,b_j\in R, u,v\in C(R)$ such that $F(x)G(x)\in nil(Q[x])$.

Case1. $F(x)G(x)=0$ or $G(x)F(x)=0$. Assume that $F(x)G(x)=0$. Since R is right Ore, there exists $b_j^{'}\in R$ and $u_1\in C(R)$ such that $u^{-1}b_j=b_j^{'}u_1^{-1}$ for $j=1,\ldots,n$. Let $f(x)=\sum_{i=0}^{m}a_ix^i$ and $g(x)=\sum_{j=0}^{n}b_j^{'}x^j$ .Then $f(x)g(x)=0$. Since R is weak McCoy, there exists $0\neq c\in R$ with $f(x)c\in nil(R[x])\subseteq nil(Q[x])$. Hence $F(x)uc=f(x)u^{-1}uc=f(x)c\in nil(Q[x])$. If $G(x)F(x)=0$, then by a similar argument we can show that $G(x)vd\in nil(Q[x])$ for some non-zero $d\in R$.

Case2. $F(x)G(x)\neq 0$ and $G(x)F(x)\neq 0$. Since $F(x)G(x)\in nil(Q[x])$, there exists $n\geq 2$ such that $(F(x)G(x))^n=0$ and $(F(x)G(x))^{n-1}\neq 0$. Let $(F(x)G(x))^n=F(x)H(x)$. If $H(x)\neq 0$, then by a similar argument as above there exists $\alpha\in C(R)$, $r\in R$ such that $F(x)\alpha r\in nil(Q[x])$. Now assume $H(x)=G(x)\underbrace{F(x)G(x)\ldots F(x)G(x)}_{n-1}=0$. Since $(F(x)G(x))^{n-1}\neq 0$ and R is weak McCoy, then by Case 1, there exists $\beta\in C(R)$, $s\in R$ such that $G(x)\beta s=0$. Therefore Q is weak McCoy.

According to Bell [2], a ring R is called semi-commutative if $ab=0$ implies $aRb=0$. We say an ideal I is a *semi-commutative ideal*, if R/I is a semi-commutative ring.

Lemma 2.9. Let R be a semi-commutative ring. If $c_1c_2\cdots c_k=0$ for some $c_i\in R$, then $c_1Rc_2Rc_3\cdots Rc_k=0$.

Proof. By induction, let $c'_{k-1}=c_{k-1}c_k$. Then $c_1c_2\cdots c'_{k-1}=0$ and by induction assumption, we have $0=c_1Rc_2Rc_3\cdots Rc'_{k-1}=c_1Rc_2Rc_3\cdots Rc_{k-1}c_k$. Hence, for all $x\in c_1Rc_2Rc_3\cdots Rc_{k-1}$, we have $xc_k=0$. It follows by hypothesis that $xRc_k=0$. Thus $c_1Rc_2Rc_3\cdots Rc_k=0$, as desired.

Lemma 2.10 (4, Lemma 2.5). Let R be a semi-commutative ring. Then $nil(R)$ is a semi-commutative ideal of R.

Proof. Let $a,b\in nil(R)$. Then $a^n=0=b^m$ for some $m,n\geq 0$. Each term of the expansion of $(a+b)^{m+n+1}$ has the form $x:=(a^{i_1}b^{j_1})\cdots(a^{i_{m+n+1}}b^{j_{m+n+1}})$ where $i_r,j_s\in N\bigcup\{0\}$. Since $(i_1+j_1)+(i_2+j_2)+\ldots+(i_{m+n+1}+j_{m+n+1})=\sum_{r=1}^{n}i_r+\sum_{s=1}^{m}j_s=m+n+1$, either $\sum_{r=1}^{n}i_r\geq n$ or $\sum_{s=1}^{m}j_s\geq m$. If $\sum_{r=1}^{n}i_r\geq n$, then $a^{i_1}a^{i_2}\cdots a^{i_{m+n+1}}=0$. Thus $(a^{i_1}b^{j_1})\cdots(a^{i_{m+n+1}}b^{j_{m+n+1}})=0$, by Lemma 2.9. If $\sum_{r=1}^{n}i_r<n$, then $\sum_{s=1}^{m}j_s\geq m$. Thus $b^{j_1}b^{j_2}\cdots b^{j_{m+n+1}}=0$ and so $(a^{i_1}b^{j_1})\cdots(a^{i_{m+n+1}}b^{j_{m+n+1}})=0$, by Lemma 2.9. Hence $(a+b)^{m+n+1}=0$.

Now suppose that $a^n=0$ and $r\in R$. Then $(ar)^n=0=(ra)^n$, by Lemma 2.9. Thus $nil(R)$ is an ideal of R.

Since $R/nil(R)$ is a reduced ring, hence it is a semi-commutative ring. Therefore $nil(R)$ is a semi-commutative ideal of R.

Lemma 2.11. Let R be a semi-commutative ring. Then $nil(R[x]) = nil(R)[x]$.

Proof. Let $f(x) = a_0 + ... + a_n x^n \in nil(R[x])$. Then $f(x)^k = 0$, for some integer $k \geq 0$. Hence $a_n{}^k = 0$, and that $a_n \in nil(R)$. There exists $g(x), h(x) \in R[x]$ such that $f(x)^k = (a_0 + ... + a_{n-1}x^{n-1})^k + a_n g(x) + h(x)a_n$. Since $nil(R)[x]$ is an ideal of $R[x]$ and $a_n g(x), h(x)a_n, f(x)^k \in nil(R)[x]$, we have $(a_0 + ... + a_{n-1}x^{n-1})^k \in nil(R)[x]$. Hence $a_{n-1}{}^k \in nil(R)$ and that $a_{n-1} \in nil(R)$. Continuing this process yields $a_0, ..., a_n \in nil(R)$. Therefore $nil(R[x]) \subseteq nil(R)[x]$.

Now, let $f(x) = a_0 + ... + a_n x^n \in nil(R)[x]$. Then $a_i^{m_i} = 0$, for some positive integer m_i. Let $k = m_0 + \cdots + m_n + 1$. Then $(f(x))^k = \sum (a_0{}^{i_{01}} (a_1 x)^{i_{11}} \cdots (a_n x^n)^{i_{n1}}) \cdots (a_0{}^{i_{0k}} (a_1 x)^{i_{1k}} \cdots (a_n x^n)^{i_{nk}})$, where $i_{0r} + \cdots + i_{nr} = 1$, for $r = 1, \cdots k$ and $0 \leq i_{rs} \leq 1$. Each coefficient of $f(x)^k$ is a sum of such elements $\gamma = ((a_0)^{i_{01}} \cdots (a_n)^{i_{n1}}) \cdots ((a_0)^{i_{0k}} \cdots (a_n)^{i_{nk}})$, where $i_{0r} + \cdots + i_{nr} = 1$.

It can be easily checked that there exists $a_k \in \{a_0, \cdots, a_n\}$ such that $i_{t1} + \cdots + i_{tk} \geq m_t$. Since $a_t{}^{m_t} = 0$ and R is semi-commutative, $\gamma = 0$. Thus $(f(x))^k = 0$ and $nil(R)[x] \subseteq nil(R[x])$. Therefore $nil(R[x]) = nil(R)[x]$.

Lemma 2.12 . Let R be a semi-commutative ring. Then $nil(R[x][y]) = nil(R[x])[y]$.

Proof. By Lemma 2.11, $nil(R[x])$ is an ideal of $R[x]$. Since $R[x]/nil(R[x])$ is a reduced ring, hence $nil(R[x])$ is a semi-commutative ideal of $R[x]$, and that $nil(R[x])[y] \subseteq nil(R[x][y])$.

Now, let $F(y) = \sum_{i=0}^{m} f_i y^i \in nil(R[x][y])$, where $f_i = \sum_{s=0}^{p_i} a_{is} x^s \in R[x]$. Then $F(y)^n = 0$, for some positive integers n. As in the proof of [1], let $k = n \sum \deg f_i$, where the degree is as polynomial in x and the degree of zero polynomial is taken to be 0. Then $(F(x^k))^n = 0$ and the set of coefficients of $F(x^k)$ is equal to the set of all coefficients of f_i, $0 \leq i \leq m$. Hence by Lemma 2.11, $a_{ij} \in nil(R)$ for all i, j and that $f_i \in nil(R[x])$, for each i. Thus $F(y) \in nil(R[x])[y]$. Therefore $nil(R[x][y]) = nil(R[x])[y]$.

If R is semi-commutative, then $R[x]$ may not be semi-commutative, by [5, Example 2]). Here we will show that if R is semi-commutative, then R is weak McCoy if and only if $R[x]$ is weak McCoy.

Theorem 2.13. If R is a semi-commutative ring, then $R[x]$ is a weak McCoy ring if and only if R is weak McCoy.

Proof. Suppose that R is a weak McCoy ring. Let $F(t)=\sum_{i=0}^{m} f_i t^i$, $G(t)=\sum_{j=0}^{n} g_j t^j$ be non-zero polynomials in $R[x][t]$ such that $F(t)G(t) \in nil(R[x][t])$, where $f_i=\sum_{s=0}^{p_i} a_{is}x^s$, $g_j=\sum_{t=0}^{q_j} b_{jt}x^t \in R[x]$. As in the proof of [1], let $k=\sum \deg f_i+\sum \deg g_j$, where the degree is as polynomial in x and the degree of zero polynomial is taken to be 0. Then $F(x^k)=\sum_{i=0}^{m} f_i x^{ik}$,$G(x^k)=\sum_{j=0}^{n} g_j x^{jk} \in R[x]$, and the set of coefficients of the $F(x^k)$ is (respectively $G(x^k)$) equal to the set of all coefficients of f_i, $0 \le i \le m$ (respectively g_j, $0 \le j \le n$). Since $(F(t)G(t))^p=0$, for some $p \ge 1$, and x commutes with elements of R, $(F(x^k)G(x^k))^p=0$. Since R is weak McCoy, there is $0 \ne r \in R$ such that $F(x^k)r \in nil(R[x])$ and $a_{is}r \in nil(R)$, $f_i r \in nil(R[x])$ for $0 \le i \le m$, $0 \le s \le p_i$ by Lemma 2.11. Hence $F(t)r \in nil(R[x][t])$, by Lemma 2.12. Therefore R[x] is weak McCoy.

Now suppose $R[x]$ is a weak McCoy ring and $f(t)g(t) \in nil(R[t]) \subseteq nil(R[x][t])$. Since $R[x]$ is weak McCoy, there exists $0 \ne h(x) \in R[x]$ such that $f(t)h(x) \in nil(R[x][t])$. Let $h(x)=a_0+\ldots+a_n x^n \in R[x]$ ($a_0 \ne 0$). Then $f(t)a_0 \in nil(R[t])$, since $(f(t)h(x))^k=(f(t)a_0)^k+k_1x+\ldots+k_{nk}x^{nk}$ with $k_1,\ldots,k_{nk} \in R[t]$. Therefore R is weak McCoy.

Theorem 2.14. Let R be a ring and Δ a multiplicatively closed subset of R consisting of central regular elements. Then R is weak McCoy if and only if $\Delta^{-1}R$ is weak McCoy.

Proof. If R is is a weak McCoy ring, then by a similar way as used in Theorem 2.8, one can show that $\Delta^{-1}R$ is weak McCoy.

Conversely, let $\Delta^{-1}R$ be a weak McCoy ring. Let $f(x)=\sum_{i=0}^{m} a_i x^i$ and $g(x)=\sum_{j=0}^{n} b_j\ x^j$ be non-zero polynomials of $R[x]$ such that $f(x)g(x)\in nil(R[x])$. Since $\Delta^{-1}R$ is weak McCoy, $f(x)(c\alpha^{-1})\in nil((\Delta^{-1}R)[x])$ for some non-zero $c\alpha^{-1}\in\Delta^{-1}R$. Thus $f(x)c\in nil(R[x])$ and R is weak McCoy.

Corollary 2.15. Let R be a ring. Then $R[x]$ is weak McCoy if and only if $R[x,x^{-1}]$ is weak McCoy.

Proof. Clearly $\Delta=\{1,x,x^2,....\}$ is a multiplicatively closed subset of $R[x]$ consisting of central regular elements and $\Delta^{-1}R[x]=R[x,x^{-1}]$. Hence the proof follows from Theorem 2.14.

Theorem 2.16. The classes of weak McCoy rings are closed under direct limits.

Proof. Let $A=\{R_i,\alpha_{ij}\}$ be a direct system of weak McCoy rings R_i for $i\in I$ and ring homomorphisms $\alpha_{ij}:R_i\to R_j$ for each $i\le j$ with $\alpha_{ij}(1)=1$, where I is a directed partially ordered set. Let $R=\varinjlim R_i$ be the direct limit of A with $\ell_i:R_i\to R$ and $\ell_j\alpha_{ij}=\ell_i$. We show that R is weak McCoy ring. Let $a,b\in R$. Then $a=\ell_i(a_i)$, $b=\ell_j(b_j)$ for some $i,j\in I$ and there is $k\in I$ such that $i\le k$, $j\le k$. Define $a+b=\ell_k(\alpha_{ik}(a_i)+\alpha_{jk}(b_j))$ and $ab=\ell_k(\alpha_{ik}(a_i)\alpha_{jk}(b_j))$, where $\alpha_{ik}(a_i)$, $\alpha_{jk}(b_j)\in R_k$. Then R forms a ring with $0=\ell_i(o)$ and $1=\ell_i(1)$. Let $f,g\in R[x]$ be non-zero polynomials such that $fg\in nil(R[x])$. There is $k\in I$ such that $f,g\in R_k[x]$. Hence $fg\in nil(R_k[x])$. Since R_k is weak McCoy, there exists $0\ne c_k\in R_k$ such that $fc_k\in nil(R_k[x])$. If $c=\ell_k(c_k)$, then $fc\in nil(R[x])$ with non-zero c. Therefore R is weak McCoy.

Proposition 2.17. (1) Let R be a ring. If there exists a non-zero ideal I of R such that $I[x]\subseteq nil(R[x])$, then R is weak McCoy.

(2) Every non-semiprime ring is weak McCoy.

(3) Let R be a ring with a non-zero nilpotent ideal. Then $Mat_n(R)$ ($n \geq 2$) is weak McCoy.

Proof. (1) Let $0 \neq f \in R[x]$. If $f \in I[x]$, then $fr \in nil(R[x])$ for all $r \in R$. If $f \notin I[x]$ then $fs \in I[x] \subseteq nil(R[x])$ for all non-zero $s \in I$. Thus R is weak McCoy.

(2) Let R be a ring with $N_*(R) \neq 0$. Since $0 \neq N_*(R)[x] = N_*(R[x]) \subseteq nil(R[x])$, R is weak McCoy by (1).

(3) Since $Mat_n(R)$ is non-semiprime, hence by (1) $Mat_n(R)$ is weak McCoy.

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